Common Fixed Point Theorem Satisfying an Integral Type Contractive Condition in Fuzzy Metric Space

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Abstract—Using a contractive condition of integral type , we have proved a fixed point theorem in fuzzy metric space with the concept of weakly compatible mappings

Index Terms—Common fixed point, weakly compatible maps,E.A.property, fuzzy metric space.

AMS subject classification : 47 H 10, 54 H 25

1 Introduction

The notion of a probabilistic metric space corresponds to the situation when we do not know the distance between points, but one knows only the possible values of the distance. Since the 16th century, probability theory has been studying a kind of uncertainty known as the randomness of the occurrence of the event. In this case, the event itself is completely certain. The study of mathematics began to explore the restricted zone fuzziness, which followed the study of uncertainty and randomness. Fuzziness is a kind of uncertainty. It is applied to these events,whose chances of occurrence are uncertain, i.e. they are in non-black or non-white state. Zadeh [16]introduced the concept of fuzzy set as a new way to represent vagueness in our everyday life.

A fuzzy set A in X is a function with domain X and values in [0, 1]. Since then, many Authors have developed a lot of literature regarding the theory of fuzzy sets and its applications.However when the uncertainty is due to fuzziness rather than randomness as in the measurement of ordinary length, it seems that the concept of fuzzy metric space is more suitable.

The first group involves those results in which a fuzzy metric on a set X is treated as map d: X xX→ R^+, where X represents the totality of all fuzzy points of a set and satisfy some axioms analogous to the ordinary metric axioms.Thus, in such an approach numerical distances are set up between fuzzy objects. On the other hand, in the second group we keep those results in which the distance between objects is fuzzy and the objects themselves may or may not be fuzzy.

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Ercez [2] have introduced the concept of fuzzy metric space in different ways. Grabiec [5] results were further generalized for a pair of commuting mappings by subramanyam [15].More over George and Veeramani [3] modified the concept of fuzzy metric spaces introduced by Kramosil and Michalek [7]. Further George and Veeramani [4] introduced the concept of Hausdorff topology on fuzzy metric space and showed that every metric induces a fuzzy metric.

The aim of this paper is to prove a common fixed points theorem satisfying an integral type inequality for weakly compatible maps using E.A. property.

This paper generalizes the results of P.P.Murthy, Sanjay Kumar,K.Tas[11] and Manish Kumar Mishra, PriyankaSarma and D.B.Ojha[9].Further this paper generalizes the metric space ofA.R.Khan and A.A. Domlo [8].

Definition 1.1: A binary operation * : [0 , 1] x [0 ,1] → [0 ,1]is called continuous t-norm if ([0 ,1] , *) iis an abelian topological monoid with unit1 such that a*b ≤ c*d

Whenever a≤ c and b ≤ d for all a,b,c,d ∈ [0 ,1].

Definition 1.2 : The triplet (X , M , *) is said to be a fuzzy metric space if X is an arbitrary set , *is a continuous t-normand M is a fuzzy set onX xXx [0 , ∞∞] ---→[0 , 1] satisfying the following conditions.

For all x,y,zϵ X and s,t > 0
1.2.1 (FM – 1) M (x,y,0) = 0
1.2.2 (FM -2) M(x,y,t) = 1 for all t > 0 if and only if x= y.
1.2.3 (FM-3) M(x,y,t) = M(y,x,t)
1.2.4 (FM-4) M(x,y,t) * M(y,z,s) ≤ M(x,z , t+s)
1.2.5(FM-5) M(x,y,.) : [0 ,1] ----→ [0 , 1] is continuous

Example 1.2.1 : [3] Let (X,d) be a metric space. Define

a*b= min { a , b } and $M(x,y,t) = \frac{t}{t+d(x,y)}$ for all x,yϵ X and all t > 0. Then (X , M , *) is a fuzzy metric space induced by d.

Definition 1.3. [4] Let f and g be two self maps of a fuzzy metric space (X , M , *). f and g are said to be compatible if M (fgx_n , gfx_n, t) ---→ 1 as n--→∞ , whenever { x_n } is a sequence in X such that fx_n , gx_n ---→ z as n ---→∞ for some t ϵ X.

Definition 1.4. [14] . The self maps f and g of a fuzzy metric space (X,M,*) are said to be occasionally weakly compatible if they commute at their coincidence point.
i.e.fgx = gfx when fx=gx , x ϵ X.

The concept of weak compatibility is most general among all the commutativity concepts, clearly each pair of compatible self maps is weakly compatible but the converse is not true.
Definition 1.5 [1] : Let f and g be two self maps of a fuzzy metric space (X,M,*). We say that f and g satisfy E.A. propertty I f there exists a sequence { x_n } in X such that

$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = x_0$ for some $x_0 \epsilon$ X.

i.e.

$\lim_{n\to\infty} M\left(fx_n, x_0,\ t\right) = 1 = \lim_{n\to\infty} M\left(gx_n, x_0,\ t\right)$

for some t ϵ X.

2. Weakly compatible mappings:

Let f and g be two self maps of a metric space (x,d) and f and g are said to be weakly commuting if

d(fgx , gfx) ≤ d(fx , gx) for every x ϵ X.

It can be shown that commuting mappings are weakly compatible , but the converse is false.

Let (X,d) be a metric space, α ϵ [0 , 1]. f : X --→X a mapping such that for every x,yϵ X

$$\int_0^{d(fx,fy)} \emptyset(t)\,dt \le \alpha \int_0^{d(x,y)} \emptyset(t)\,dt$$

Where $\emptyset$: $R^+ --\to R$ is a lebesgue integral mapping which is summable , ϵ> 0 , $\int_0^{\epsilon} \emptyset(t)$ dt > 0 is non negative for some ϵ> 0. Then f has unique common fixed point z ϵ X such that for each x,

$\lim_{n\to\infty} f^n x$ = z for some z ϵ X. Rhoades [13] extend this result by replacing the above condition by the following :

$$\int_0^{d(fx,fy)} \emptyset(t)dt \le \alpha \int_0^{\max\{ d(x,y),d(x,fx),d(y,fy),\frac{1}{2}\{d(x,fy)+d(y,fx)\}\}} \emptyset(t)dt$$

Ojha et al [12] extended this result to the multi valued mappings and replaced the above condition by the following :

$$\int_0^{d^p_{(Fx,Fy)}} \emptyset(t)dt \le \int_0^{\max\left\{ad(fx,fy).d^{p-1}_{(fx,Fx)},ad(fx,fy).d^{p-1}_{(fx,Fy)},ad(fx,Fx)d^{p-1}_{(fx,Fy)},cd^{p-1}_{(fx,Fy)}d(fy,Fx)\right\}} \emptyset(t)dt$$

For all x,yϵ X, where p ≥ 2 is an integer a ≥ 0 and 0 < c < 1. Then f and F have a unique common fixed point in X.

Theorem 2.1

Let f,g be two weak compatible self maps of a fuzzy metric space (X,M,*) satisfying the E.A.property and
(2.1.1) f(X) ⊆ g (X)

(2.1.2) $\int_0^{M(fx,fy,kt)} \emptyset(t)dt \ge \int_0^{M(gx,gy,t)} \emptyset(t)dt$

(2.1.3) $\int_0^{M(fx,fy,t)} \emptyset(t)dt >$

$$\int_0^{\min\{M(fx,gy,t),\gamma M(fx,gx,t),\alpha M(fy,gy,t),\frac{1}{2}\{M(fx,gy,t)+M(gy,fx,t)\}} \emptyset(t)dt$$

Where $\gamma\epsilon$ [0 , ∞) , α ϵ [0 ,1) and fx≠ fy.

If the range of f or g is a complete subspace of X then f and g have a unique common fixed point,

Proof:

Since f and g satisfy E.A. property , so there exists a sequence {x_n} in X such that fx_n--→z , gx_n--→z as n--→ ∞ for some z ϵ X.

(2.1.4) $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$ for some z ϵ X.

As zϵ f(X) and f(X) ⊂ g(X). Assume that g(X) is a complete subspace of X then there exists some point u ϵ X such that gx_n----→z where z=gu for some u ϵ X.

If fu≠gu then

(2.1.5) $\int_0^{M(fx_n,fu,kt)} \emptyset(t)dt \ge \int_0^{M(gx_n,gu,kt)} \emptyset(t)dt$

Taking limit as n ---→∞ we get

(2.1.6) $\int_0^{M(gu,fu,kt)} \emptyset(t)dt \ge \int_0^{M(gu,gu,kt)} \emptyset(t)dt$

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It is a contradiction. Hence fu=gu. This tells that f and g have coincidence point, since f and g are weakly compatible maps so fgu = gfu when fu=gu.

So fgu=ffu=gfu=ggu.

Suppose fu≠ ffu then by (2.1.3) we have

$\int_0^{M(fu,ffu,t)} \emptyset(t)dt>$

$$\int_0^{\min\{M(fu,gfu,t),\gamma M(fu,gu,t),\alpha M(ffu,gfu,t),\frac{1}{2}\{M(fu,gfu,t)+M(ffut,gu,t)\}} \emptyset\,(t)dt$$

$\int_0^{M(fu,ffu,t)} \emptyset(t)dt>\int_0^{M(fu,ffu,t)} \emptyset(t)dt$which is a contradiction.

There forefu = ffu andfu=ffu = fgu = gfu = ggu

Hence fu is the common fixed point of f and g.
Hence z is a common fixed point of f and g.

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